Test 2A - MTH 1310

Dr. Graham-Squire, Spring 2017

Name: Coy

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(signature)

DIRECTIONS

1. Don't panic.
2. Show <u>all of your work</u> and <u>use correct notation</u>. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first 3 questions of the test, however you should still show all of your work. No calculators are allowed on the last 3 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 10. Total Points = 65.

(... points) Use calculus to find the absolute maximum and absolute minimum

$$g(x) = \frac{x}{16 + x^2}$$

on the interval $[-5, 3]$. Make sure to show your work!

$$g'(x) = \frac{(16+x^2)\cdot 1 - x(2x)}{(16+x^2)^2}$$ ✓✓

$$= \frac{16+x^2-2x^2}{(16+x^2)^2}$$

$$0 = \frac{16-x^2}{(16+x^2)^2}$$ ✓

$\Rightarrow 16-x^2=0$

$(4-x)(4+x)=0$ ✓

$\Rightarrow x=4$ or $x=-4$

Not in interval

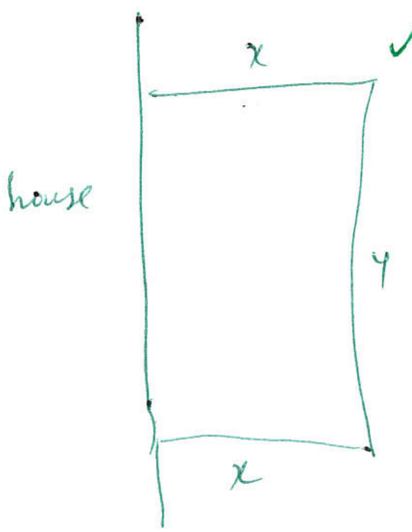
check:

$f(-5) = \frac{-5}{41} = -0.121$

$f(-4) = \frac{-4}{32} = -.125$ ← abs min ✓

$f(3) = \frac{3}{25}$ ← Abs. Max ✓

2. (8 points) Michael wants to build a rectangular pen ... only needs to build 3 sides eating tiger. Since the pen is next to his house, Michael only needs to build 3 sides to the pen–two sides will be perpendicular to the house, and the other side parallel. Since the parallel side is visible to his neighbors, he wants to make that side out of nice cider pickets, at a cost of \$8 per linear foot. The other two sides he will make out of a cheaper steel fencing which only costs \$5 per running foot. If he has \$500 to spend on building the pen, what are the dimensions that will give a maximum area for the pen? *Use calculus* to find the answer, and make sure to show your work! Round your answer(s) to the nearest 0.1 foot.



$8y + 5(2x) = 500 \Rightarrow \frac{500 - 8y}{10} = \frac{10x}{10}$

$50 - 0.8y = x$

Area $= xy$

Area $= (50 - 0.8y)y$

$A(y) = 50y - 0.8y^2$

$A'(y) = 50 - 1.6y$

$0 = 50 - 1.6y$

$1.6y = 50$

$$\boxed{\begin{array}{l} y = \frac{50}{1.6} = 31.25 \\ x = 50 - 0.8(31.25) = 25 \end{array}}$$

... Mary are both considering investing some money at a 6% intere... rate. Assume that the investment will be continuously compounded.

(a) If Bob invests \$1000, how long will it take until he has \$1500? Round your answer to the nearest 0.1 year.

(b) How much would Mary need to invest initially if she wanted to end up with \$3000 after investing for 4 years? Round to the nearest cent.

$A = Pe^{rt}$ ✓

$r = 0.06$

(a) $1500 = 1000e^{0.06t}$ ✓

✓ $\ln 1.5 = \ln e^{0.06t}$

$\frac{\ln 1.5}{0.06} = t = \boxed{6.8 \text{ years}}$ 0.5

(b)

$3000 = Pe^{0.06(4)}$ ✓

✓ $\frac{3000}{e^{0.24}} = P = \boxed{\$2359.88}$ 0.5

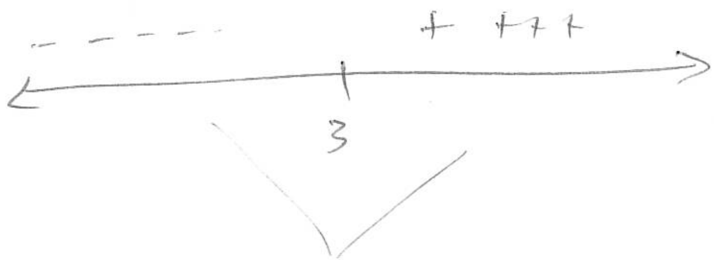
4. (6 points) Use calculus to find the x-value of the relative maximum of $f(x) = e^{(x-3)^2}$, if it exists. If it does not exist, write NONE. You can use a graphing calculator to double-check your answer, but you must use calculus and show your work to receive full credit.

✓✓ $f'(x) = e^{(x-3)^2}(2(x-3)\cdot 1)$

✓ $0 = e^{(x-3)^2}(2)(x-3)$

$\underset{\neq 0}{\downarrow} \qquad \underset{\neq 0}{\downarrow} \qquad x-3=0 \Rightarrow x=3$

✓✓



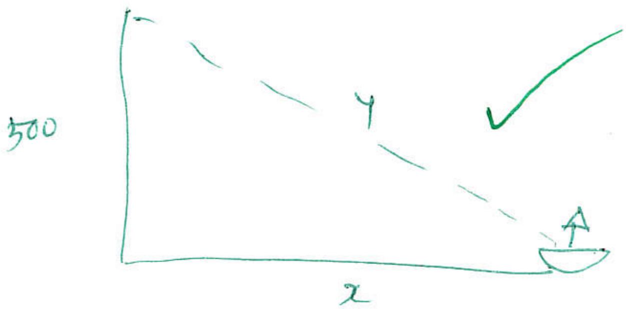
$f'(4) = +$

$f'(0) = -$

✓ $\Rightarrow x = 3$ is a relative minimum, there is no relative max $\Rightarrow$ $\boxed{\text{NONE}}$

Round to nearest 0.1 ft/min

5. (6 points) Helen is at the top of a 500 foot cliff overlooking the ocean, watching a on the water coming toward her. Suppose the boat is moving at a constant speed 100 feet/minute on the water. At what rate is the (diagonal) distance between Helen and the boat changing when the boat is 2000 feet from the base of the cliff?

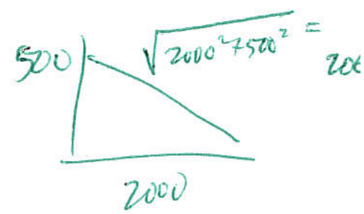


$\frac{dx}{dt} = -100$

0.5

$\frac{dy}{dt} = ?$ when $x = 2000$

$\frac{d}{dt}\left(x^2 + 500^2 = y^2 \right)$



0.5

$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$

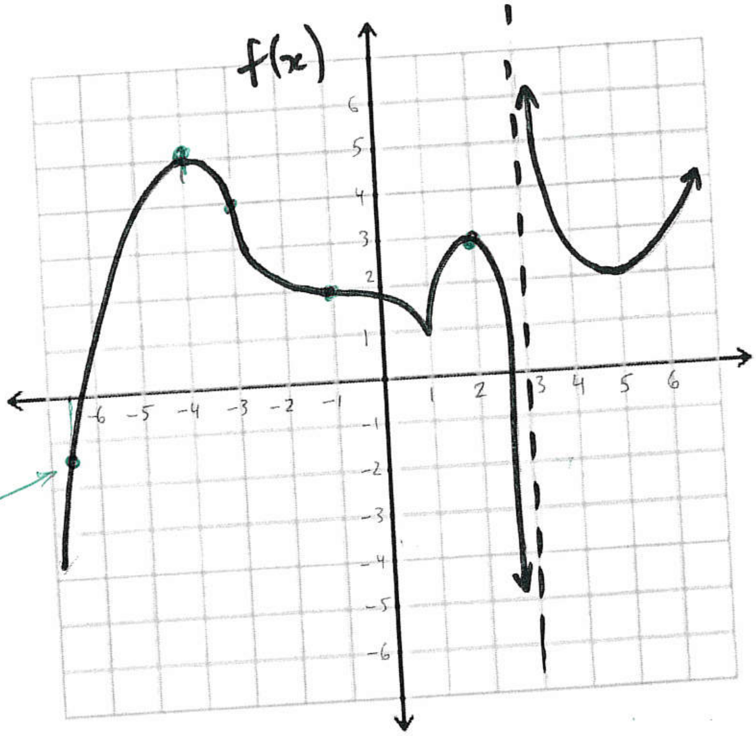
$2(2000)(-100) = 2(2061.55) \frac{dy}{dt}$

$\frac{-400000}{2(2061.55)} = \frac{dy}{dt}$

$\Rightarrow \boxed{-97.01 \text{ ft/min}}$

97.0 also okay

6. (8 points) For the graph ... necessary, but may ... item exists, put DNE. Explanations are not necessary, but may ... your answer is wrong.



abs. Min.

-0.5 if pts.

2

(a) The interval(s) where $f(x)$ is decreasing, if any exist.

$\{ (-4,-1)(-1,1)\ (2,3)\ (3,5)$

(-4, 1) okay

1

(b) The (x, y) coordinates where f has relative maximum(s), if any exist.

(-4, 5) and (2, 3)

2

(c) The interval(s) where f is concave up, if any exist.

$(-3,-1)\quad (3,\infty)$

2

(d) The (x, y) coordinates of the inflection point(s) of f, if any exist.

(-3, 4) (-1, 2)

1

(e) The absolute minimum value of f on the interval $[-6.5, 2]$, if it exists.

2 $\boxed{-1.5}$

NO CALCULATOR

Name: Key

7. (8 points) Let $f(x) = \frac{1}{3}x^3 - 5x^2 + 16x - 7$. Find the following, and make sure to show your work to receive full credit! If any of the answers do not exist, write NONE.

(a) The interval(s) where $f(x)$ is increasing, if any exist.

(b) At what x-value(s) f has any relative minimums, if any exist.

(c) The interval(s) where f is concave down, if any exist.

(d) The x-value(s) of the inflection point(s) of f, if any exist.

(a) $f'(x) = x^2 - 10x + 16$ ✓

0.5 $0 = (x-8)(x-2)$

$+$ $-$ $+$

2 8 ✓

$f'(0) = +$

$f'(4) = -$

$f'(10) = +$

(a) increasing on $(-\infty, 2)$, $(8, \infty)$ ✓

(b) relative min @ $\boxed{x = 8}$ 0.5

(c) $f''(x) = 2x - 10$ ✓

$-$ $+$ ✓

0.5 $0 = 2(x-5) \Rightarrow x = 5$

5

$f''(4) = -$

$f''(6) = +$

(c) Concave down on $(-\infty, 5)$ ✓

(d) infl. pt @ $\boxed{x = 5}$ 0.5

8. (5 points) Use logarthmic differentiation to calculate the derivative of

$$y = \frac{\sqrt{x} \cdot (x^2 - 3)^7}{e^{x^3}}$$

f don't
o log...

You do not need to simplify your answer.

$$\ln y = \ln\left(\frac{x^{1/2} \cdot (x^2-3)^7}{e^{x^3}}\right)$$

0.5

$$\ln y = \ln x^{1/2} + \ln (x^2-3)^7 - \ln e^{x^3}$$ ✓

$$\frac{d}{dx}\left(\ln y = \frac{1}{2}\ln x + 7 \ln (x^2-3) - x^3\right)$$ ✓

✓✓ $$\frac{y'}{y} = \frac{1}{2} \cdot \frac{1}{x} + 7\left(\frac{1}{x^2-3}\right) \cdot 2x - 3x^2$$

0.5 $$\boxed{y' = \left(\frac{1}{2x} + \frac{14x}{x^2-3} - 3x^2\right)\left(\frac{\sqrt{x}\ (x^2-3)^7}{e^{x^3}}\right)}$$

Max of 4 for direct.

9. (6 points) Find the function $f(x)$ if $f'(x) = 18x^8 + 5e^x$ and $f(0) = 3$.

$$f(x) = \int (18x^8 + 5e^x)\,dx$$ ✓✓

$$f(x) = 18\left(\frac{x^9}{9}\right) + 5e^x + C$$ ✓✓

$$f(x) = 2x^9 + 5e^x + C$$

$$f(0) = 2(0)^9 + 5e^0 + C$$ ✓

$\Downarrow$

$$3 = 0 + 5 + C$$

$$-2 = C$$ ✓

$$\Rightarrow \boxed{f(x) = 2x^9 + 5e^x - 2}$$

10. (6 points) Calculate the antiderivative. Show your work for full points:

$$\int -30x^{-3}\left(\frac{3}{x^2}+7\right)^4 dx$$

$u = 3x^{-2} + 7$

$\frac{du}{dx} = -6x^{-3}$

$du = -6x^{-3}\,dx$

$\frac{du}{-6x^{-3}} = dx$

$= \int -30\cancel{x^{-3}}(u^4)\left(\frac{du}{-6\cancel{x^{-3}}}\right)$

$= \int 5u^4\,du$

$= u^5 + C$

$= \boxed{\left(\frac{3}{x^2}+7\right)^5 + C}$

0.5 0.5

Extra Credit(up to 3 points) You can choose to have either 1 extra credit point or 3 extra credit points. If you choose 1 point, you are guaranteed to get 1 point. If you choose 3 extra credit points, and 3 or more students in the class (including yourself) choose 3 points, then everyone who chose 3 points will get NO extra credit.

01

1 pt	3 pts
𝍸 III	𝍸 I

02

1 pt	3 pts
𝍸 𝍸 𝍸 I	